Supplementary Material

Estimating the Distribution of Sensorimotor Synchronization Data: A Bayesian Hierarchical Modeling Approach

Rasmus Bååth

One advantage of working within a Bayesian statistical framework is that it is relatively straight forward to modify and extend model definitions. What follows are four possible modifications of the models presented in the main paper. Example implementation of these model using R and JAGS can be found here: https://github.com/rasmusab/bayes_timing

1. Extending the Model with Informative Priors

One advantage with a Bayesian approach is that prior knowledge about task performance can be incorporated into the analysis. A subjective Bayesian model is feasible in the case where there exists sufficient prior information regarding the parameters. This is often the case regarding SMS studies where many published papers include descriptive statistics of the distribution of constant error and timing variability at different tempi (for a comprehensive review see Repp and Su, 2013; Repp, 2005). Prior information can be incorporated in the hierarchical model by replacing the vague top-level priors with distributions that can be made more or less informative depending on the strength of the prior information. What follows is a modification of the hierarchical model presented in the paper that enables the inclusion of prior information in the analysis.

The prior on the group mean μ_μ is a normal distribution with parameters $\mu_{\mu,\mu}$ and $\sigma_{\mu,\mu}$. The prior on the mean group SD m_σ is a log-normal distribution with parameters $\mu_{m.\sigma}$ and $\sigma_{m,\sigma}$. To facilitate the use of informative priors this prior is reparameterized to be specified by its arithmetic mean $m_{m,\sigma}$ and SD $s_{m,\sigma}$. As proposed by Gelman, 2006, the SD parameters s_σ and σ_σ are given half-Cauchy priors with parameter $s_{s,\sigma}$ and $s_{\sigma,\mu}$ respectively. It is straightforward to be informative regarding the half-Cauchy priors as the scale parameter defines the median of the distribution (Lunn et al., 2012). The full specification of model is then:

$$Y_{ij} \sim \text{Right-Cenc-Normal}(\mu_j, \sigma_j, c_j)$$
$$\mu_j \sim \text{Normal}(\mu_\mu, \sigma_\mu)$$
$$\sigma_j \sim \text{Log-Normal}(\mu_\sigma, \sigma_\sigma)$$
$$\mu_\mu \sim \text{Normal}(\mu_{\mu,\mu}, \sigma_{\mu,\mu})$$
$$\sigma_\mu \sim \text{Half-Cauchy}(s_{\sigma,\mu})$$
$$\mu_\sigma = \log(m_\sigma) - \sigma_\sigma^2/2$$
$$\sigma_\sigma = \sqrt{\log(s_\sigma^2/m_\sigma^2 + 1)}$$
$$m_\sigma \sim \text{Log-Normal}(\mu_{m,\sigma}, \sigma_{m,\sigma})$$
$$s_\sigma \sim \text{Half-Cauchy}(s_{s,\sigma})$$
$$\mu_{m,\sigma} = \log(m_{m,\sigma}) - \sigma_{m,\sigma}^2/2$$
$$\sigma_{m,\sigma} = \sqrt{\log(s_{m,\sigma}^2/m_{m,\sigma}^2 + 1)}$$

Figure 1 shows a hierarchical diagram of the model specification.

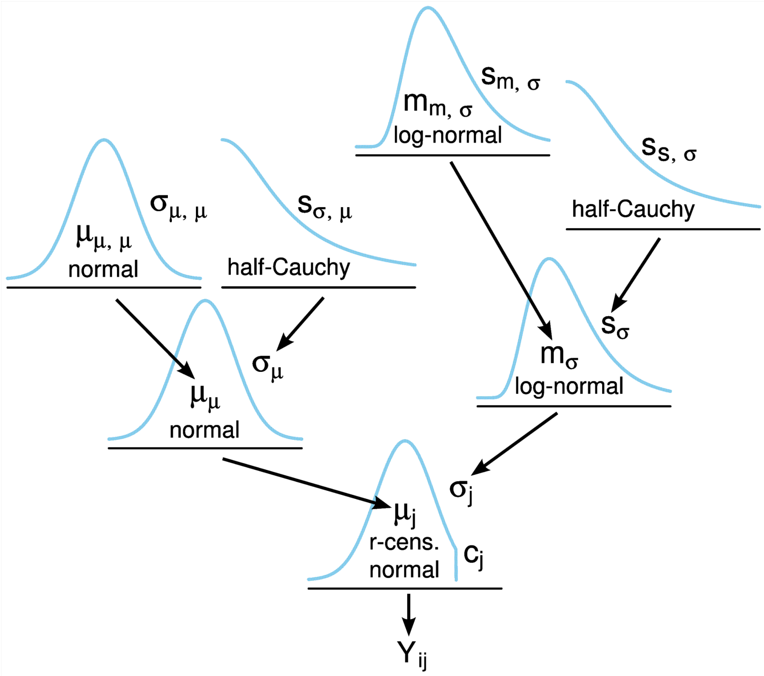

Figure 1
A diagram of the informative hierarchical model.

2. Extending the Model by Adding a Functional Dependency Between ISI levels.

The hierarchical model presented in the paper currently allows that data from one participant informs the parameters of all other participants due to the hierarchical structure of the model. Data from one ISI level does not inform parameters for other ISI levels, however. A dependency between ISI levels can be introduced in many ways, where one possibility is to introduce a functional dependency between ISI levels for the parameters at the group level. Below is a modification of the hierarchical model where there group mean (μ_μ) and the group standard deviation (m_σ) is assumed to depend linearly on the ISI level. Here k indexes the different ISI levels with ISI_k being the ISI in ms at level k.

$$
\begin{aligned}
&Y_{i,j,k} \sim \text{Right-Cenc-Normal}(\mu_{j,k}, \sigma_{j,k}, c_{j,k})\\
&\mu_{j,k} \sim \text{Normal}(\mu_{\mu,k}, \sigma_{\mu,k})\\
&\sigma_{j,k} \sim \text{Log-Normal}(\mu_{\sigma,k}, \sigma_{\sigma,k})\\
&\mu_{\mu,k} = \beta_{\mu,0} + \text{ISI}_k \cdot \beta_{\mu,ISI}\\
&\sigma_{\mu,k} \sim \text{Uniform}(0, \text{ISI}_k/2)\\
&\mu_{\sigma,k} = \log(m_{\sigma,k}) - \sigma_{\sigma,k}^2/2\\
&\sigma_{\sigma,k} = \sqrt{\log(s_{\sigma,k}^2/m_{\sigma,k}^2+1)}\\
&m_{\sigma,k} = \beta_{\sigma,0} + \text{ISI}_k \cdot \beta_{\sigma,ISI}\\
&s_{\sigma,k} \sim \text{Uniform}(0, \text{ISI}_k/4)
\end{aligned}
$$

Where the regression coefficients $\beta_{\mu,0}$, $\beta_{\mu,ISI}$, $\beta_{\sigma,0}$, $\beta_{\sigma,ISI}$ could be given vague priors. Care has to be taken so that $m_{\sigma,k}$ will not take negative values. This can be done by shifting the ISI values so that the shortest ISI level is at the zero and constraining $\beta_{\sigma,0}$, $\beta_{\sigma,ISI}$ to take on only positive values. As an example, data from twelve participants from Bååth and Madison (2012) was used to fit this model. Figure 2 and Figure 3 show the median posterior for the mean asynchrony and asynchrony SD. The colored circles show the group mean ($\mu_{\mu,k}$ in green, $m_{\sigma,k}$ in red), with the colored bars showing one and two SDs ($\sigma_{\mu,k}$ in green, $s_{\sigma,k}$ in red), and the gray circles showing the estimates for each participant. In the case where the assumed functional dependency between ISI levels corresponds well with the data then this modification of the hierarchical model will allow for better informed estimates than if data from each ISI level was estimated on its own.

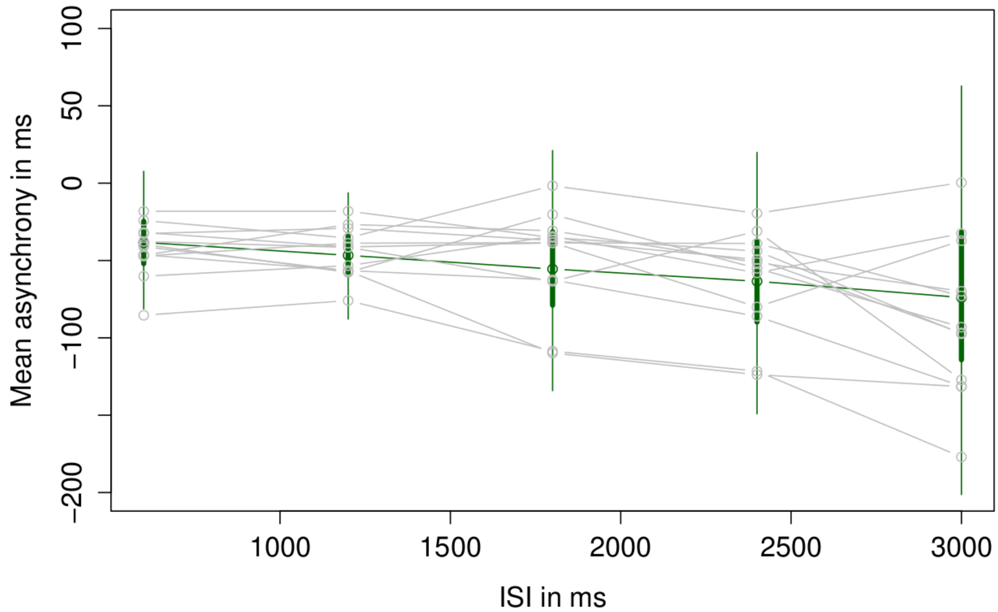

Figure 2
Estimated group mean (green) with individual estimates in grey from the hierarchical model with a functional dependency between ISI levels.

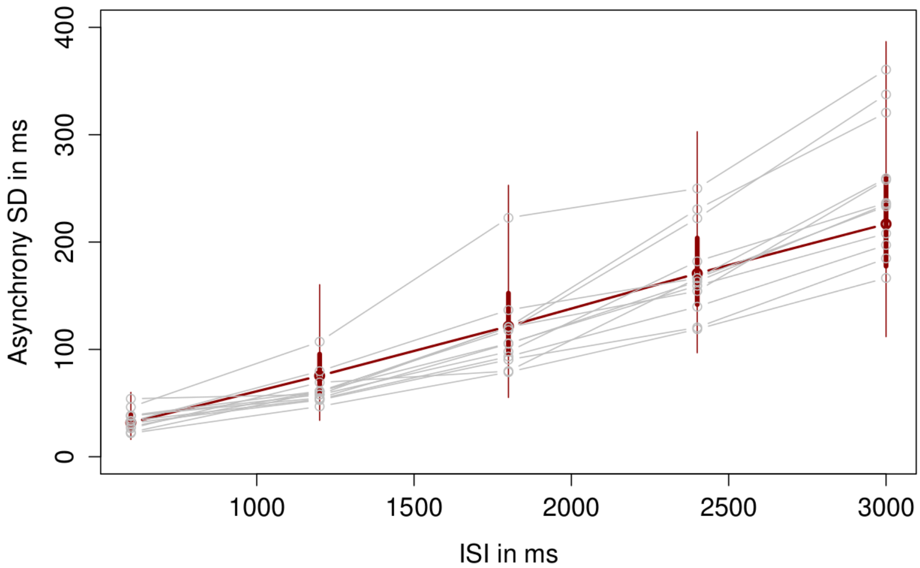

Figure 3
Estimated group SDs (red) with individual estimates in grey from the hierarchical model with a functional dependency between ISI levels.

3. Extending the Model by Modeling the Correlation between Timing Performance at Different ISI levels.

An alternative way of introducing dependencies between ISI levels is to model parameters at the participant level as coming from a multivariate normal distribution. This type of dependency can capture patterns such as that participants with large timing variability at the 600 ms ISI level also tend to have a relatively large variability at the 1200 ms ISI level. This, without imposing a functional dependency between ISI levels. Here there many options, one or more participant level parameters could be given a multivariate normal distribution, and the parameters of the multivariate normal distributions could in turn be assumed to dependent on the ISI level. Below is a modification of the hierarchical model presented in the paper where the logarithms of the participant level asynchrony standard deviations, $\log(\sigma_{j,k})$, are modeled as being distributed as a multivariate normal distribution. Here k again indexes the different ISI levels with ISI_k being the ISI in ms at level k, and n being the total number of ISI levels. Now $\mu_{\sigma,1\ldots n}$ is a vector of means and Σ_σ is an n by n covariance matrix. A default non-informative prior to use for Σ_σ could be an Inverse-Wishard distribution with parameters I_n, a n by n identity matrix, and n degrees of freedom.

$$
\begin{aligned}
&Y_{i,j,k} \sim \text{Right-Cenc-Normal}\left(\mu_{j,k}, \sigma_{j,k}, c_{j,k}\right)\\
&\mu_{j,k} \sim \text{Normal}\left(\mu_{\mu,k}, \sigma_{\mu,k}\right)\\
&\log\left(\sigma_{j,k}\right) \sim \text{Multi-Normal}\left(\mu_{\sigma,1..n}, \Sigma_\sigma\right)\\
&\mu_{\mu,k} \sim \text{Uniform}\left(-\text{ISI}_k/2, \text{ISI}_k/2\right)\\
&\sigma_{\mu,k} \sim \text{Uniform}\left(0, \text{ISI}_k/2\right)\\
&\mu_{\sigma,k} \sim \text{Uniform}\left(\log(1), \log\left(\text{ISI}_k/2\right)\right)\\
&\Sigma_\sigma \sim \text{Inv-Wishard}\left(I_n, \text{df}: n\right)
\end{aligned}
$$

Figure 4 shows posterior draws from the $\text{Multi-Normal}\left(\mu_{\sigma,1..n}, \Sigma_\sigma\right)$ distribution that resulted from fitting the model above to finger tapping data from the 30 participants in Bååth and Madison (2012). There is some positive correlation visible, indicating that participants that had a high variability at one ISI level tended to have a relatively high variability at other ISI levels. This correlation also seems strongest between adjacent ISI levels.

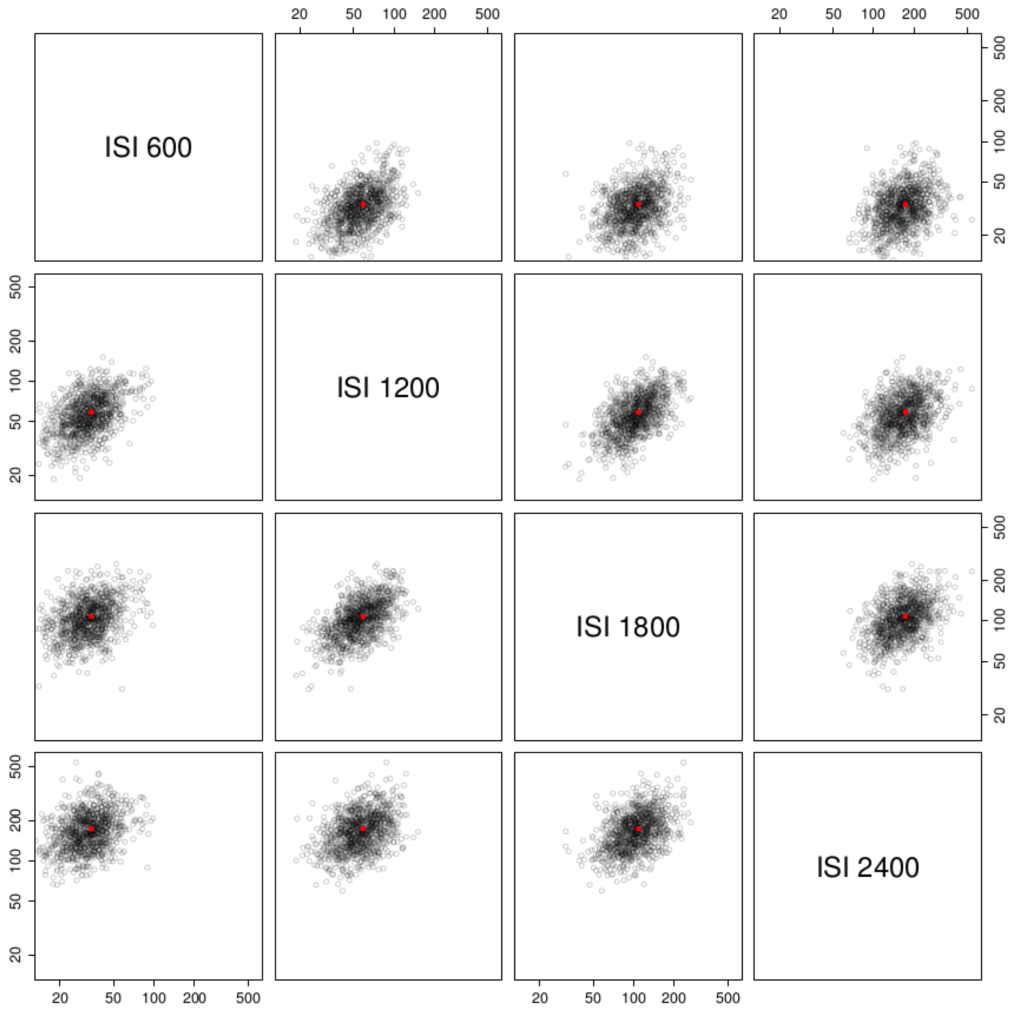

Figure 4

The estimated correlation structure shown as a sample of 1000 draws from the posterior distribution of the $\text{Multi-Normal}(\mu_{\sigma,1..n}, \Sigma_\sigma)$ distribution. The red squares marks the marginal means of the posterior.

4. Extending the Model to work with Interresponse Intervals.

The model described in this paper model the stimulus-to-response asynchronies in an SMS task. When the timing responses are self-paced, as in the synchronization-continuation paradigm (Stevens, 1886), there is no referent tone onset and the interresponse intervals, the time difference between consecutive timed responses, are instead the focus of the analysis. The model described in the this paper can be modified to accommodate interresponse interval data by changing the right-censored normal distribution to a normal distribution and by modifying the prior distribution parameters. For the non-hierarchical model a proposal would be to use:

$$I_i \sim \text{Normal}(\mu,\sigma)$$
$$\mu \sim \text{Uniform}(T/k, k\cdot T)$$
$$\log(\sigma) \sim \text{Uniform}\left(\log(1), \log\left(\frac{k}{2}\cdot T\right)\right)$$

Where T is the target interval and k is a constant that is large enough so that the prior distributions include all reasonable values of μ and σ.

References

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Lunn, D., Jackson, C., Best, N., Thomas, A., & Spiegelhalter, D. (2012). *The BUGS book: A practical introduction to Bayesian analysis*. Chapman and Hall/CRC.

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Stevens, L. T. (1886). On the time-sense. *Mind*, 11(43), 393 – 404.