

An Oscillator Model of Categorical Rhythm Perception

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Abstract

Categorical perception is a well studied phenomenon in, for example, colour perception, phonetics and music. In this article we implement a dynamical systems model of categorical rhythm perception based on the resonance theory of rhythm perception developed by Large (2010). This model is used to simulate the categorical choices of participants in two experiments of Desain and Honing (2003). The model is able to accurately replicate the experimental data. Our results supports that resonance theory is a viable model of rhythm perception and they show that by viewing rhythm perception as a dynamical system it is possible to model properties of categorical perception.

Keywords: Categorical perception; rhythm perception; dynamical systems; resonance theory

Introduction

Categorical perception occurs when categorization is amplified by the perceptual systems so that distances within a category are perceived as being smaller and distances between categories are perceived as larger than they are according to the values of some physical measurement. It is a common phenomenon that is well studied in, for example, colour perception, phonetics and music (Harnad, 1990). A central question for understanding categorical perception is: What is the underlying mechanism? There have been attempts to model categorical perception in terms of neural networks (Damper & Harnad, 2000). In this article, we will focus on modelling categorical rhythm perception and present a model that is based on oscillators.

In the field of music perception *rhythm* refers to a temporal pattern of sound onsets. A rhythm in this sense does not have to be periodic or recurrent. This is in contrast with how that word is used in other fields (cf. circadian rhythm or delta rhythm). A related concept that does involve periodicity is *beat*. When listening to a piece of music a common response is to move one's body with a perceived periodic pulse (Snyder & Krumhansl, 2001), that pulse is the beat of the corresponding piece of music. It is not common that all beats in a piece of music are perceived as being equally accented and a periodically recurring pattern of strong and weak beats is called a *meter*. For example, a duple meter would imply that every second beat is perceived as having a stronger accent while every third beat is perceived as having a stronger accent in the case of a triple meter. Rhythm perception and the ability to entrain to a musical beat was long thought to be uniquely human and, while it has recently been shown that some vocal mimicking species are, to some degree, able to move in synchrony with a beat (Schachner, Brady, Pepperberg, & Hauser,

2009), humans are still unique in their aptitude for rhythmic processing. Already infants have been shown to have a sense of rhythm (Honing, Ladinig, Háden, & Winkler, 2009) and there exists only one documented case of "beat deafness" (Phillips-Silver et al., 2011), that is, the inability to reliably synchronize to a musical beat.

Desain and Honing (2003) showed in two experiments that listeners reliably experienced rhythms as belonging to rhythmic categories and that categorizations were strongly influenced when the listeners were primed with a metric beat before hearing a rhythm. Furthermore, participants agreed to a large degree on which rhythms belonged to what category and, similar to categorization of other kinds of stimuli (c.f. Jäger (2010) on colour categories), the categories were found to be roughly convex with respect to a temporal performance space (Gärdenfors, 2000). Honing (2013) concludes that: "It is puzzling, however, that although meter was shown to exert a strong influence on the recognition of rhythm [...] existing computational models of meter can explain this phenomenon only to a small extent". In this article we show that an oscillation based, *resonance theory* model of rhythm perception (Large, 1996, 2010) can replicate many of the findings of Desain and Honing (2003). Our results support the notion that resonance theory is a viable model of rhythm perception and show that by viewing rhythm perception as a dynamical system it is indeed possible to model the properties of categorical rhythm perception.

Resonance Theory and Categorical Rhythm Perception

Modelling of human timing and rhythm perception has a long history. One influential model is the one described by Wing and Kristofferson (1973), which is based on an information theoretic perspective. Like many such models (cf. Repp, 2005), it models a participant's behaviour when attempting to elicit isochronous timing responses. An alternative to this information theoretic perspective is to take a dynamical systems perspective and model time and rhythm perception as an emergent, dynamic phenomenon. A number of models of this kind have been proposed (e.g., Large, 1996; Noorden and Moelants, 1999). Here, the term *resonance theory* (cf. Large, 2010) will be used to refer to this type of models. Resonance theory does not dictate a specific model but rather incorporates a number of related models. All can be considered as dynamical system models and they consist of one or more resonating oscillatory units. Resonance theory provides a compelling framework since it is biologically plausible, has

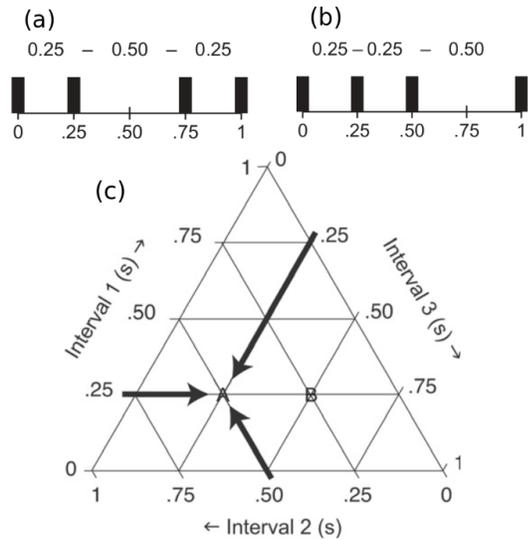


Figure 1: (a) and (b) show examples of two possible rhythms and their placement in the triangular performance space (c) defined by Desain and Honing (2003). All one second long, four sound rhythms can be represented as a point in this space. (From Honing, 2013 with permission).

a solid base in dynamical systems theory and is able to model many aspects of meter and rhythm perception (Large, 2000). The general idea of resonance theory is that an external auditory rhythm can be represented by the rhythm of internal oscillatory units. These oscillatory units are coupled to the external rhythm and are by definition periodic while the external rhythm does not have to be periodic. Given a rhythm sequence as input the basic output of a resonance theory model, or resonance model for short, is the amplitude response of the oscillators over time. This high dimensional representation might be difficult to work with directly, however, and a more convenient representation is given by creating an *activation pattern*, A , by summing the amplitude responses of each oscillator over time, as in

$$A_i = \sum_{t=t_s}^{t_e} a_{i,t} \quad (1)$$

where $a_{i,t}$ is the amplitude for oscillator i at time t while t_s and t_e are the start and end time steps for the summation. Before the resonance model is given any input it is in a resting state and it takes a number of time steps before the system is activated by the stimuli. Therefore it is not necessarily desirable to sum over the whole extent of the duration of the rhythm sequence and an activation pattern created by summing over the latter time steps may represent the rhythm sequence better than an activation pattern created by summing over all time steps.

While not all resonance theory models claim biological plausibility, a number of neuroimaging studies have shown connections between neural resonance and rhythm perception

(e.g., Brochard, Abecasis, Potter, Ragot, and Drake, 2003). One persuasive study that clearly showed that rhythm perception involves neural oscillatory activity is that of Nozaradan, Peretz, Missal, and Mouraux (2011). They found that playing a rhythmic beat to a participant elicited a sustained periodic neural response, as measured by EEG, that matched the frequencies of the beat. Resonance theory models differ in their biological plausibility, the number of oscillatory units employed and the type of oscillators used. Eck (2002) constructed a model with a clear biological connection as it used the Fitzhugh-Nagumo model of neural action potential (Nagumo, Arimoto, & Yoshizawa, 1962) as the oscillatory unit. Other models claim no biological plausibility, for example, the model by Scheirer (1998) that employs a comb filter as the oscillatory unit. Toiviainen and Snyder (2003) modelled participants behaviour when tapping along to excerpts of music composed by Bach using a single oscillatory unit while Large and Kelso (2002) used a bank of 96 oscillators to model participants' tapping to ragtime music.

To our knowledge, resonance theory models have not previously been used to model categorical rhythm perception. One reason for this might be that while the amplitude response of the oscillators in the resonance model reflects, perhaps even represents, the rhythm sequence given as input to the system it does not give rise to a categorization *per se*. That is, while the state the resonance model arrives at depends on the given rhythm sequence, there is no finite number of discrete states that can be said to constitute categories. Still, the state of the resonance model can be used as the basis of a categorical decision based on learnt prototype states or a discrete partitioning of the system state space. By considering the activation pattern of a resonance model as a point in an n -dimensional space, n being the number of oscillatory units, this space can be partitioned into regions corresponding to rhythm categories and used to produce categorical decisions (following the general model of concepts from Gärdenfors (2000)). The relation between the activation pattern of a resonance model and such a rhythm categorization is analogous to the relation between the hue, saturation and lightness of a colour percept and a colour categorization. That is, a colour percept can similarly be viewed as a point in a three dimensional space with dimensions hue, saturation and lightness and this space can be partitioned into regions, each representing a colour category, and used to produce categorical colour decisions.

If the state of the resonance model is viewed as the basis for a categorical decision then two predictions regarding categorical rhythm perception can be made:

(1) More *distinct* states will facilitate categorization. Here a distinct state refers to a subset of oscillators in a resonance model having a strong amplitude response while most oscillators show a weak amplitude response. This is in contrast to a non-distinct state where most oscillators have a similar amplitude response, that is, there are many competing signals and there is no clear single winning candidate among

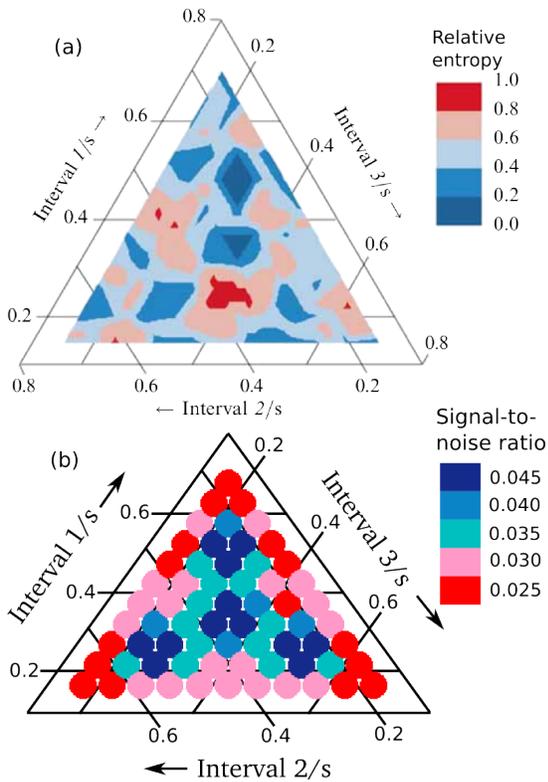


Figure 2: Maps over categorization consistency. (a) shows the entropy of the categorical choices for the participant given the same rhythm sequences multiple times from Desain and Honing (2003, used with permission). (b) shows the signal-to-noise measure calculated from the activation patterns generated by the resonance model.

the categories. In an experimental categorical task it would then be predicted that a participant would categorize a rhythm sequence more consistently, and with more confidence, if the sequence resulted in a more distinct state in a resonance model than if the sequence resulted in a less distinct state.

(2) Rhythm sequences resulting in similar states are categorized similarly. That is, different rhythm sequences resulting in similar states when used as the input to a resonance model should be categorized similarly by participants in an experimental task.

In order to test these predictions, data from a rhythm categorization task is needed. A study by Desain and Honing (2003) provides suitable experimental data to do so.

The Rhythm Categorization Study of Desain and Honing (2003)

Desain and Honing (2003) employed a novel paradigm where participants were asked to categorize 66 different rhythm sequences by transcribing them into common music notation. The sequences all lasted for one second and consisted of four tone onsets and were therefore uniquely determined by the three interonset intervals (IOI) between the tones. Two such

possible sequences are shown in figure 1a and 1b where a possible categorization of 1b could be ♪♪♪ (or 1-1-2 when written as an integer ratio). Any possible one second, four tone rhythm sequence can be thought of as a point in a three dimensional triangular *performance space* that determines the lengths of the three IOIs as shown in figure 1. The 66 rhythm sequences were constructed so that they evenly covered the area in the performance space with the constraint that no IOI would be shorter than 153 ms. The location of these sequences in the performance space can be seen in figure 2b where each circle marks the position of one of the 66 sequences.

In a first experiment, 29 highly trained musicians categorized the rhythm sequences and the result was that even though the rhythms were performed on a more or less continuous scale, the participants tended to stick to a limited number of categories with 1-1-1 being the single most common. Twelve categories, all categories considered, stood out as being the most common and the location in performance space of these categories are shown in figure 3a. One participant was presented with the 66 rhythm sequences multiple times and as a measure of consistency the entropy was calculated of her responses for each rhythm. These entropy values were mapped on to the performance space and the resulting entropy map is shown in figure 2a.

In a second experiment two meter conditions were added. Duple meter versions of the rhythms were constructed by prepending the rhythms with a repeated, one second long, two sound beat, thus inducing a 2/4 meter feeling. Triple meter versions of the rhythms were similarly constructed by prepending a three sound beat instead. This resulted in three different meter conditions: The original no meter condition, a duple meter condition and a triple meter condition. Maps over what categories the participants ascribed to the different rhythms, similar to the map shown in figure 3a, were constructed (shown on p. 358 in Desain and Honing, 2003). A main finding was that the participants' categorization in the no meter condition was significantly more similar to the participants' categorization in the duple meter condition than in the triple meter condition.

For the purpose of the current study, data from Desain and Honing was downloaded from a web resource containing supplementary material¹. The data downloaded was the information regarding which of the twelve most common categories was most often ascribed to each of the 66 rhythm sequences for the no meter condition in experiment one and the duple and triple meter conditions in experiment two. A data point for a rhythm sequence was excluded if none of the twelve most common categories was the most common for that specific rhythm. Information regarding the categorization entropy for the participant presented with the rhythms multiple times was unfortunately not available from the web resource. This information was retrieved manually from figure 2a.

¹<http://www.mcgv.uva.nl/categorization>

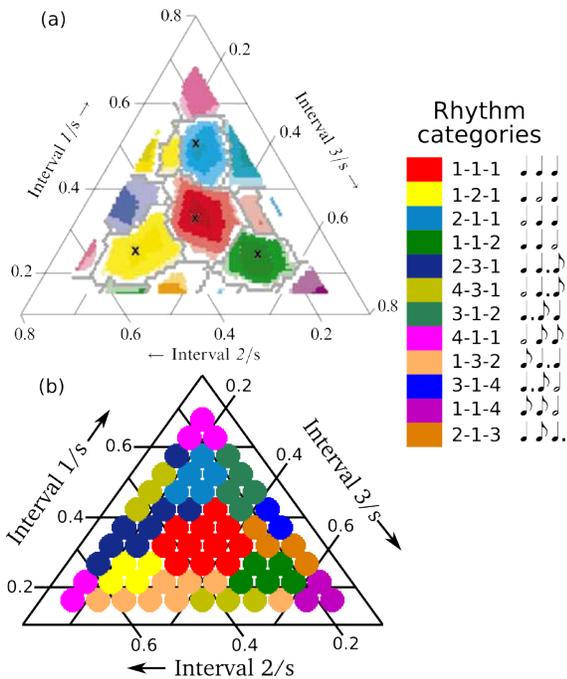


Figure 3: Categorization maps for (a) the experimental data from Desain and Honing (2003, used with permission) and (b) the resonance model. The transparent areas in (a) indicate areas where there was a large amount of disagreement between the participants.

Resonance Theory and the Data of Desain and Honing

It is possible to test the two predictions from resonance theory concerning how rhythms are categorized by implementing a resonance model that consists of an array of oscillators (as in Large, 2000). We used the rhythm stimuli from Desain and Honing (2003) as input to such a model and compared the results with the experimental data using the methods outlined below.

Prediction (1) implies that rhythm sequences resulting in distinct states in a resonance model should be the sequences that are categorized more consistently. In Desain and Honing’s data, a measure of consistency is the categorization entropy for the participant presented with the rhythm sequences multiple times. The prediction is that this measure of consistency is correlated with a measure of distinctness of the state of a resonance model. Signal-to-noise ratio is a common measure of distinctness of a signal and a modified version of this measure can be used to quantify the distinctness of the state of a resonance model. For a resonance model that has been given a rhythm sequence as input, the activation pattern is first calculated according to equation (1). In this activation pattern, the signal A_s is defined as being the A_i with the highest amplitude. The signal-to-noise ratio is then defined

as:

$$SNR = \frac{A_s}{\sum_{i=1}^n A_i} \quad i \neq s \quad (2)$$

where the sum in the denominator is over the rest of the A_i oscillator amplitudes. Notice that this measure of consistency should be negatively correlated with the entropy measure of Desain and Honing: As the signal gets weaker relative to the noise, the entropy of the participants choices of category should go up.

Prediction (2) implies that rhythm sequences resulting in similar states when given as input to a resonance model should be categorized similarly in an experimental task. A resonance model does not directly produce a categorization but this is not required for testing this prediction. It is possible to compare the resulting states of two rhythm sequences by calculating the respective activation patterns and comparing these. A suitable similarity measure is given by considering the activation patterns as points in an n -dimensional space, where n is the number of oscillators in the resonance model, and then taking the Euclidean distance between these two points, where a shorter distance corresponds to more similar states. Considering the twelve most common rhythm categories chosen by the participants in Desain and Honing’s study as prototype categories, it is possible to use the rhythm sequences corresponding to these categories to generate *prototype activation patterns*. For example, to generate the prototype activation pattern for the category 1-2-1 (as shown in figure 4) the rhythm sequence with IOIs 0.25 s, 0.5 s and 0.25 s would be used as input to the resonance model. A rhythm sequence’s activation pattern can then be compared with these prototypes’ activation patterns and the prototype category with the most similar activation pattern can be assigned to that rhythm sequence. In this way, all rhythm sequences can be assigned a category and these categories can be compared with the categories selected by the participants in Desain and Honing’s study. Specific hypotheses are then that a resonance model categorization of the stimulus used by Desain and Honing should be similar to the categorization made by the participants in the no meter, duple meter and triple meter conditions. Furthermore, since the participants’ categorizations of the rhythm sequences in the duple meter condition was more similar than the triple meter condition to the categorization in the no meter condition the same relation should be present in the categories generated by the resonance model.

The Setup of the Resonance Model

The resonance model was implemented in MATLAB² using the Nonlinear Time-Frequency Transformation Workbench (Large and Velasco, in preparation). The model consisted of 145 *Hopf oscillators*, a type of oscillator that entrains to periodic input and where the amplitude of an oscillator depends on that oscillator’s intrinsic frequency and the periodicities

²<http://www.mathworks.se/products/matlab/>

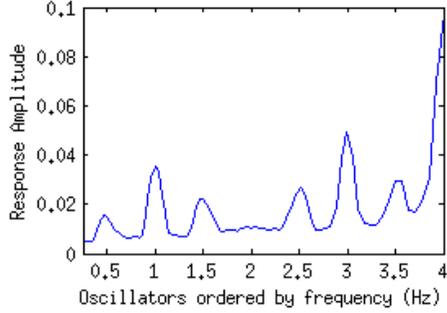


Figure 4: An example of an activation pattern generated by feeding the resonance model a rhythm with IOIs 0.25 s, 0.5 s and 0.25 s.

of the input. The differential equation of the Hopf oscillator used is:

$$\frac{dz}{dt} = z \left(\alpha + i\omega + \frac{\beta \varepsilon |z|^4}{1 - \varepsilon |z|^2} \right) + \frac{x}{1 - \sqrt{\varepsilon} x} \cdot \frac{1}{1 - \sqrt{\varepsilon} z} \quad (3)$$

$$\alpha = -0.1, \quad \beta = -0.1, \quad \varepsilon = 0.5$$

where α is a damping term, β is an amplitude compression factor and ε is a scale factor. The last term in equation (3) is the resonant term, which is dependent on the stimulus x . These parameter values and this specific formulation of the Hopf oscillator were not chosen on the basis of any specific theoretical considerations; many other configurations are possible and a more general form of the Hopf oscillator is derived in Large, Almonte, and Velasco (2010). The oscillators' intrinsic frequencies were centred around 1 Hz with frequencies logarithmically distributed from 0.25 Hz to 4 Hz. The method used for creating activation patterns was that in equation (1) with t_s set to the time step corresponding to half the stimulus length and t_e set to the last time step³.

The 66 rhythm sequences from the no meter condition were encoded and given as input to the model yielding 66 activation patterns. This was repeated for the rhythm sequences from the duple and triple meter conditions. Additionally, the rhythm sequences of the prototype categories were encoded in the same way as the no meter condition sequences yielding twelve prototype activation patterns.

Results

The signal-to-noise measure was calculated for all activation patterns in the no meter condition and, as predicted, a negative correlation between Desain and Honing's (2003) entropy measure of consistency and the signal-to-noise ratio (Pearson product-moment correlation, $r = -0.33$, $p = 0.006$) was found. The two measures of consistency are expected

³The MATLAB code for the model and both input data and the resulting output are available on request from the first author. The code for the Nonlinear Time-Frequency Transformation Workbench (Large and Velasco, in preparation) has not yet been publicly released and has to be requested separately.

to have a reverse relationship, that is, low entropy in the experimental data should indicate high consistency, while a low signal-to-noise ratio in the simulated data should indicate low consistency. A comparison between these two measure of consistency is shown in figure 2. To facilitate comparison, the colour scales have been matched so that red indicates low consistency while blue indicates high consistency. The measures of consistency are clearly comparable, showing the same broad patterns in both the simulated (figure 2b) and experimental data (figure 2a).

The activation patterns for all the three meter conditions were compared with the prototype activation patterns using Euclidean distance as the similarity measure and each rhythm sequence was assigned the category of the most similar prototype. A comparison with the categories assigned in the experimental task for the no meter condition is shown in figure 3. It is clear that the categorizations to a large extent agree. The 1-1-1 category is the most common in both the experimental and the simulated categorizations and both categorizations exhibit roughly convex category regions. A randomized permutation test⁴ also showed that the categorization generated by the resonance model and the categorization from Desain and Honing's data was more similar than would be expected by chance alone for all the three meter conditions. In the no meter condition (shown in figure 3) the agreement was 71% ($p < 0.001$) and in the duple and triple meter conditions 67% ($p < 0.001$) and 61% ($p < 0.001$) respectively.

In the experimental data, the categorization of the duple meter condition was more similar than the triple meter condition to the no meter condition and this was also the case for the simulated categorizations. The agreement between the no meter condition and the duple and triple meter conditions for the simulated categorizations was calculated as being 77% and 71% respectively with duple meter agreeing with the no meter categorization in 6 percentage points more of the cases ($p = 0.045$, one-tailed randomized permutation test).

Conclusions

Many models of categorical perception have been based on neural networks and there exist several models of rhythm perception based on neural networks (Mozer, 1993; Miller, Scarborough, & Jones, 1992). We believe that using a dynamical

⁴Randomized permutation tests (Ernst, 2004) were used to compare the categorization of the rhythm sequences from the behavioural data with the categorization from the resonance model. Given two different categorizations of the 66 rhythms a similarity score, is calculated as the number of rhythms that are given the same category by both categorizations. In the cases when the most common categorization of a specific rhythm sequence in the behavioural data is not one of the twelve prototype categories this rhythm sequence is excluded from further analysis. Next, all category labels are randomly assigned to different rhythm sequences and a new similarity score is calculated. This is repeated 10,000 times, yielding a randomized permutation distribution of similarity scores. A p-value is then calculated as the probability of achieving the actual similarity score, or a similarity score being more extreme, given the randomized distribution of similarity scores. The permutation tests were two-tailed (calculated according to the method in Ernst, 2004) in all cases except when noted.

system of resonating oscillators provides a more principled way of modelling such phenomena. By modelling rhythm perception in such a system, we have shown that it is possible to explain empirical findings of listeners' categorical perception of rhythm. Our oscillator model has been able to accurately replicate the experimental data from Desain and Honing (2003).

An advantage of oscillator models is that they can be generalized to other kinds of categorical perception. Examples from the domain of music are pitch perception and tonality perception (Large, 2010). Oscillatory models are not confined to temporal processes and can be used for other modalities. The main importance of our model is perhaps that the example of how oscillator models can be constructed for categorical rhythm perception can serve as inspiration for similar models of other cognitive phenomena.

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References

- Brochard, R., Abecasis, D., Potter, D., Ragot, R., & Drake, C. (2003). The "ticktock" of our internal clock : direct brain evidence of subjective accents in isochronous sequences. *Psychological Science, 14*(4), 362–366.
- Damper, R., & Harnad, S. (2000). Neural network models of categorical perception. *Attention, Perception, & Psychophysics*.
- Desain, P., & Honing, H. (2003). The formation of rhythmic categories and metric priming. *Perception, 32*(3), 341–365.
- Eck, D. (2002). Finding downbeats with a relaxation oscillator. *Psychological research, 66*(1), 18–25.
- Ernst, M. D. (2004). Permutation methods: a basis for exact inference. *Statistical Science, 19*(4), 676–685.
- Gärdenfors, P. (2000). *Conceptual Spaces: The Geometry of Thought*. MIT press.
- Harnad, S. (1990). *Categorical Perception: The Groundwork of Cognition*. Cambridge University Press.
- Honing, H. (2013). Structure and interpretation of rhythm in music. In D. Deutsch (Ed.), *The psychology of music* (pp. 369–390). Elsevier.
- Honing, H., Ladinig, O., Háden, G. P., & Winkler, I. (2009). Is beat induction innate or learned? Probing emergent meter perception in adults and newborns using event-related brain potentials. *Annals of the New York Academy of Sciences, 1169*, 93–6.
- Jäger, G. (2010). Natural color categories are convex sets. In *Logic, language and meaning: 17th amsterdam colloquium, amsterdam*. Springer.
- Large, E. W. (1996). Modeling beat perception with a nonlinear oscillator. In *Proceedings of the Eighteenth Annual Conference of the Cognitive Science Society: July 12-15, 1996, University of California, San Diego* (p. 420). Lawrence Erlbaum.
- Large, E. W. (2000). On synchronizing movements to music. *Human Movement Science, 19*(4), 527–566.
- Large, E. W. (2010). Neurodynamics of music. In M. Riess Jones, R. R. Fay, & A. N. Popper (Eds.), *Music Perception* (pp. 201–231). Springer.
- Large, E. W., Almonte, F. V., & Velasco, M. J. (2010). A canonical model for gradient frequency neural networks. *Physica D: Nonlinear Phenomena, 239*(12), 905–911.
- Large, E. W., & Kelso, J. A. S. (2002). Tracking simple and complex sequences. *Psychological Research, 3*–17.
- Large, E. W., & Velasco, M. (n.d.). *Nonlinear Time-Frequency Transformation Workbench: MATLAB software for signal processing using nonlinear oscillators*.
- Miller, B., Scarborough, D., & Jones, J. (1992). On the perception of meter. *Understanding Music with AI*.
- Mozer, M. (1993). Neural net architectures for temporal sequence processing. In A. Weigend & N. Gershenfeld (Eds.), *Predicting the future and understanding the past*. (pp. 243–264). Addison-Wesley.
- Nagumo, J., Arimoto, S., & Yoshizawa, S. (1962). An active pulse transmission line simulating nerve axon. *Proceedings of the IRE, 50*(10), 2061–2070.
- Noorden, L. van, & Moelants, D. (1999). Resonance in the perception of musical pulse. *Journal of New Music Research, 28*(1), 43–66.
- Nozaradan, S., Peretz, I., Missal, M., & Mouraux, A. (2011). Tagging the neuronal entrainment to beat and meter. *The Journal of Neuroscience : The Official Journal of the Society for Neuroscience, 31*(28), 10234–40.
- Phillips-Silver, J., Toivianen, P., Gosselin, N., Piché, O., Nozaradan, S., Palmer, C., et al. (2011). Born to dance but beat deaf: A new form of congenital amusia. *Neuropsychologia, 49*(5), 961–969.
- Repp, B. H. (2005). Sensorimotor synchronization: A review of the tapping literature. *Psychonomic Bulletin & Review, 12*(6), 969.
- Schachner, A., Brady, T., Pepperberg, I., & Hauser, M. (2009). Spontaneous motor entrainment to music in multiple vocal mimicking species. *Current Biology, 19*(10), 831–836.
- Scheirer, E. D. (1998). Tempo and beat analysis of acoustic musical signals. *The Journal of the Acoustical Society of America, 103*(1), 588–601.
- Snyder, J., & Krumhansl, C. L. (2001). Tapping to Ragtime: Cues to Pulse Finding. *Music Perception, 18*(4), 455–489.
- Toivianen, P., & Snyder, J. (2003). Tapping to Bach: Resonance-based modeling of pulse. *Music Perception, 21*(1), 43–80.
- Wing, A., & Kristofferson, A. (1973). Response delays and the timing of discrete motor responses. *Attention, Perception, & Psychophysics, 14*(1), 5–12.